Online Supplementary Material for

Women's College Decisions: How Much Does Marriage Matter?

by

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Abstract

In this document, I use a simple two-period example to illustrate analytically how various sources determine college decisions, discuss the related empirical issues, and examine the identification of the model. To confront the data, many assumptions are relaxed in the empirical model specified in the paper.

1 An Illustrative Model

Let us consider a sample of high school women, the size of which is normalized to one. Each woman lives for two periods and is endowed with ability δ, where $\delta \in \{\delta_h, \delta_l\}$ and $\delta_h > \delta_l$. A fraction π of the sample belongs to high skill type δ_h.

In the first period, each woman stays single and makes her decision to attend college (and graduate). Attending college requires a fixed cost cs, and none of the women work while in college. In the second period, each woman works full-time. If a woman works, her labor earnings y_{it} take the form of $\ln y_{it} = \beta_0(\delta_i) + \beta_1 S_{it} + \epsilon_{wit}$, where $S_{it} \in \{1, 2\}$ denotes high school and college, respectively. The skill rental price, β_0, increases with individual ability. That is, $\beta'(\delta) > 0$. The schooling coefficient β_1 measures the earnings benefit of college. Productivity shocks ϵ_{wit} are assumed to be *i.i.d.* normal, serially uncorrelated, with mean zero, variance σ_w^2, and *c.d.f.* $F_w(\cdot)$.

In the second period, women decide whether or not to get married. A marriage is formulated only if a woman receives and accepts an offer from a man. Assume that an infinite number of men (either high school or college graduates) exist in the economy and that a proportion μ of them are college graduates. The meeting technology is such that the marriage offer probabilities may be different for college and high school women. Let P^1 and P^0 be the marriage offer arrival rates for college and high school women, respectively.[1] The probability of a type S woman receiving a marriage proposal from a type S^H man, where

[1] Empirical estimation of marriage offer probabilities in the paper shows that $P^1 > P^0$.

$S, S^H \in \{1, 2\}$, is determined by

$$\left\{\begin{array}{lll} P^0(1-\mu) & \text{if} & \left(S, S^H\right) = (1,1) \\ P^0\mu & \text{if} & \left(S, S^H\right) = (1,2) \\ P^1(1-\mu) & \text{if} & \left(S, S^H\right) = (2,1) \\ P^1\mu & \text{if} & \left(S, S^H\right) = (2,2) \end{array}\right\}.$$

Let M_{it} be the net utility value of marriage and $M_{it} = a_0 + a_1(S_{it} - S_{it}^H)^2$. The value of marriage depends on the couple's homogeneity in educational background to capture educational assortative matching.[2] A married woman can consume a fraction ψ of the husband's earnings, which depend on his schooling $\left(S^H\right)$ and follow $\ln y_i^H = \rho_0 + \rho_1 S_i^H + \epsilon_{Hi}$, where $\epsilon_{Hi} \sim N\left(0, \sigma_H^2\right)$. Therefore, the schooling level of the husband increases the marriage payoff for the woman.

The utility is separable in consumption and the value of marriage: $U_{it} = c_{it} + M_{it}m_{it}$. If a woman is married, $m_{it} = 1$; otherwise, $m_{it} = 0$. Each woman solves the following problem:

$$Max_{\{s_{i1}, m_{i2}\}} E[c_{i1} + \beta(c_{i2} + M_{i2}m_{i2})]$$

$$\begin{aligned} s.t.\quad c_{i1} + cs \cdot s_{i1} &\leq (1 - s_{i1})y_{i1} \\ c_{i2} &\leq y_{i2} + \psi y_{i2}^H m_{i2}, \end{aligned}$$

where s_{it} equals 1 if attendance is chosen and 0 otherwise, and β is the discount rate. The expectation is taken over the distribution of the woman's own earnings, marriage offer probability, and the potential husband's earnings.

2 Determinants of College Decisions

The model is solved backwards. At $t = 2$, alternative-specific value functions conditional on female's schooling S_{i2} and male's schooling S_{i2}^H can be written as follows:

$$\begin{aligned} &V_{i2}(m_{i2}=1; S_{i2}, S_{i2}^H) = y_{i2} + \psi \exp\left(\rho_0 + \rho_1 S_{i2}^H + \epsilon_{Hi2}\right) + a_0 + a_1(S_{i2} - S_{i2}^H)^2, \\ &V_{i2}(m_{i2}=0; S_{i2}) = y_{i2}, \qquad\qquad S_{i2} = 1, 2,\ S_{i2}^H = 1, 2. \end{aligned}$$

A woman marries if and only if $V_{i2}(m_{i2} = 1; S_{i2}, S_{i2}^H) \geq V_{i2}(m_{i2} = 0; S_{i2})$. Since the husband's earnings are nonnegative, a woman i always marries if the marriage value $M_{i2} = a_0 + a_1(S_{i2} -$

[2] In this simple example, there is no uncertainty in the marriage value M.

$S_{i2}^H)^2 \geq 0$. When $M_{i2} < 0$, she marries if and only if

$$\epsilon_{Hi2} \geq \ln \frac{-a_0 - a_1(S_{i2} - S_{i2}^H)^2}{\psi} - \left(\rho_0 + \rho_1 S_{i2}^H\right). \tag{1}$$

Denote

$$\epsilon_H^*\left(S_{i2}, S_{i2}^H\right) \equiv \ln \frac{-a_0 - a_1(S_{i2} - S_{i2}^H)^2}{\psi} - \left(\rho_0 + \rho_1 S_{i2}^H\right).$$

At $t = 1$, the value of attending college is

$$\begin{aligned}
V_{i1}(s_{i1} = 1) &= -cs + \beta E \max[V_{i2}(m_{i2} = 1; 2, S_{i2}^H), V_{i2}(m_{i2} = 0; 2)] \\
&= -cs + \beta \int_{-\infty}^{\infty} \{\mu P^1 [\int_{\epsilon_H^*(2,2)}^{\infty} V_{i2}(m_{i2} = 1; 2, 2)\, dF(\epsilon_{Hi2}) + \int_{-\infty}^{\epsilon_H^*(2,2)} V_{i2}(m_{i2} = 0; 2)\, dF(\epsilon_{Hi2})] \\
&\quad + (1 - \mu) P^1 [\int_{\epsilon_H^*(2,1)}^{\infty} V_{i2}(m_{i2} = 1; 2, 1)\, dF(\epsilon_{Hi2}) + \int_{-\infty}^{\epsilon_H^*(2,1)} V_{i2}(m_{i2} = 0; 2)\, dF(\epsilon_{Hi2})] \\
&\quad + \left(1 - P^1\right) V_{i2}(m_{i2} = 0; 2)\} dF(\epsilon_{wi2}).
\end{aligned}$$

The current value of college attendance is the net cost of cs. The expected future value includes three terms: if the woman receives an offer from a college man (with probability μP^1), the future utility depends on the marriage decision and the cut-off level $\epsilon_H^*(2, 2)$; if the woman receives an offer from a high school man (with probability $(1 - \mu) P^1$), the future utility depends on the cutoff level $\epsilon_H^*(2, 1)$; if no marriage offer is received (with probability $(1 - P^1)$), the future utility is the value of being single. The expectations are also taken over woman's own wage distributions. Similarly the value of not attending college is

$$\begin{aligned}
V_{i1}(s_{i1} = 0) &= y_{i1} + \beta E \max[V_{i2}(m_{i2} = 1; 1, S_{i2}^H), V_{i2}(m_{i2} = 0; 1)] \\
&= y_{i1} + \beta \int_{-\infty}^{\infty} \{\mu P^0 [\int_{\epsilon_H^*(1,2)}^{\infty} V_{i2}(m_{i2} = 1; 1, 2)\, dF(\epsilon_{Hi2}) + \int_{-\infty}^{\epsilon_H^*(1,2)} V_{i2}(m_{i2} = 0; 1)\, dF(\epsilon_{Hi2})] \\
&\quad + (1 - \mu) P^0 [\int_{\epsilon_H^*(1,1)}^{\infty} V_{i2}(m_{i2} = 1; 1, 1)\, dF(\epsilon_{Hi2}) + \int_{-\infty}^{\epsilon_H^*(1,1)} V_{i2}(m_{i2} = 0; 1)\, dF(\epsilon_{Hi2})] \\
&\quad + \left(1 - P^0\right) V_{i2}(m_{i2} = 0; 1)\} dF(\epsilon_{wi2}).
\end{aligned}$$

Individual i attends college if and only if

$$V_{i1}(s_{i1} = 1) \geq V_{i1}(s_{i1} = 0).$$

When the expected future value of college is large enough, the college attendance condition is characterized by the following cut-off rule: a woman attends college if and only if

$$\epsilon_{wi1} \leq \epsilon_w^*, \tag{2}$$

where

$$\begin{aligned}
\epsilon_w^* = \ln\{ & -cs + \beta \int_{-\infty}^{\infty} \{\mu P^1 [\int_{\epsilon_H^*(2,2)}^{\infty} V_{i2}(m_{i2} = 1; 2, 2)\, dF(\epsilon_{Hi2}) + \int_{-\infty}^{\epsilon_H^*(2,2)} V_{i2}(m_{i2} = 0; 2)\, dF(\epsilon_{Hi2})] \\
& - \mu P^0 [\int_{\epsilon_H^*(1,2)}^{\infty} V_{i2}(m_{i2} = 1; 1, 2)\, dF(\epsilon_{Hi2}) + \int_{-\infty}^{\epsilon_H^*(1,2)} V_{i2}(m_{i2} = 0; 1)\, dF(\epsilon_{Hi2})] \\
& + (1-\mu) P^1 [\int_{\epsilon_H^*(2,1)}^{\infty} V_{i2}(m_{i2} = 1; 2, 1)\, dF(\epsilon_{Hi2}) + \int_{-\infty}^{\epsilon_H^*(2,1)} V_{i2}(m_{i2} = 0; 2)\, dF(\epsilon_{Hi2})] \\
& - (1-\mu) P^0 [\int_{\epsilon_H^*(1,1)}^{\infty} V_{i2}(m_{i2} = 1; 1, 1)\, dF(\epsilon_{Hi2}) + \int_{-\infty}^{\epsilon_H^*(1,1)} V_{i2}(m_{i2} = 0; 1)\, dF(\epsilon_{Hi2})] \\
& + \left(1 - P^1\right) V_{i2}(m_{i2} = 0; 2) - (1 - P^0) V_{i2}(m_{i2} = 0; 1)\} dF(\epsilon_{wi2})\} - \beta_0 - \beta_1.
\end{aligned} \tag{3}$$

ϵ_w^* is a function of the parameter vector of the model, θ. The parameters include π, cs, $\beta_0(\delta_h)$, $\beta_0(\delta_l)$, β_1, σ_w, P^0, P^1, a_0, a_1, ψ, ρ_0, ρ_1, σ_H.[3] College attendance rate in the economy is

$$\Pr(S = 2) = F_w(\epsilon_w^*(\theta)). \tag{4}$$

As equation (4) shows, when a woman makes her college attendance decision in the first period, not only does she take the cost of college into account, she also takes into account both future earning expectations and future marriage expectations. This equation is the key structural equation to be used to estimate how much a college decision is determined by various sources. A reduced form equation will be an approximation of equation (4). For example, the college attendance probability can be written as a probit of the cost of college, some proxy for ability, earnings gains, and marriage gains. Thus, reduced form coefficient estimates are functions of the fundamental parameters of the model. Attendance decision is determined by all costs and benefits as embedded in $\epsilon_w^*(\theta)$. Some comparative statics are stated in the proposition below.

Proposition 1 *The probability of attending college decreases in direct cost of college, and increases in marriage offer rate of college educated women. That is,* $\partial\epsilon_w^*(\theta)/\partial cs < 0$ *and* $\partial\epsilon_w^*(\theta)/\partial P^1 > 0$.

Proof. *It follows Equation (3) immediately that* $\partial\epsilon_w^*(\theta)/\partial cs < 0$ *and* $\partial\epsilon_w^*(\theta)/\partial P^1 > 0$. ■

How ability (through β_0), earnings return to schooling (β_1), and marriage sorting (a_1)

[3] It is well known that β is not well identified, so it is given. Furthermore, the focus is on women's decisions, so μ is also exogenously determined.

affect college attendance are theoretically ambiguous.[4] Once the parameters are estimated, the decision rules of the model, i.e., equations (1)–(3), will predict who attends college and how individuals adjust their behavior if the cost and benefit of attending college changes. By setting $\beta_1 = 0$ or $a_1 = 0$, I can compute the change in the threshold level and measure the counterfactual effect of earnings benefits and assortative mating on the decision to attend college.

3 Identification

In general, the nonlinearity makes it difficult to establish theoretical identification. One way to think about identification is that, as a necessary condition, each parameter should affect some moments in the distribution.

Let us first consider a homogeneous sample where $\beta_0(\delta_i) = \beta_0$ for all δ_i. Women's earnings parameters are identified from a cross-section OLS regression on $\ln y_{i2} = \beta_0 + \beta_1 S_{i2} + \epsilon_{wi2}$, since everyone works in the second period and schooling is predetermined.

Denote Π_S as the attendance rate and $\Pi_m(S, S^H)$ as the proportion of married women whose own schooling is S and whose husband's schooling is S^H. When $M_{i2} < 0$, the model implies the following moment condition:

$$\Pi_m(1,1) = (1 - \Pi_S)(1 - \mu) P^0[1 - F_H(\ln(-\frac{a_0}{\psi}) - \rho_0 - \rho_1)]. \tag{5}$$

A high school couple is observed if a woman does not attend college (with probability $(1 - \Pi_S)$) and if she receives an offer from a high school man (with probability $(1 - \mu) P^0$) and accepts the offer (i.e. $\epsilon_{Hi2} \geq \ln(\frac{-a_0}{\psi}) - \rho_0 - \rho_1$). Similarly,

$$\Pi_m(1,2) = (1 - \Pi_S)\mu P^0[1 - F_H(\ln(-\frac{a_0 + a_1}{\psi}) - \rho_0 - 2\rho_1)], \tag{6}$$

$$\Pi_m(2,1) = \Pi_S (1 - \mu) P^1[1 - F_H(\ln(-\frac{a_0 + a_1}{\psi}) - \rho_0 - \rho_1)], \tag{7}$$

$$\Pi_m(2,2) = \Pi_S \mu P^1[1 - F_H(\ln(-\frac{a_0}{\psi}) - \rho_0 - 2\rho_1)]. \tag{8}$$

In equations (5) to (8), Π_m's and Π_S are observed and μ is exogenously given. The model is not identified since there are four equations and eight unknowns: $\{P^0, P^1, a_0, a_1, \psi, \rho_0, \rho_1, \sigma_H\}$. Data on the husband's schooling and earnings provide additional moments. The conditional

[4] As β_0 and β_1 increase, college attendance may increase because expected earnings increase. But college attendance may also decrease because forgone earnings in the first period also increase. Since normally people work for many years, the first effect dominates. The effect of the sorting parameter a_1 on college attendance will depend on the schooling distribution of potential husbands.

mean and the variance of the husband's earnings can be written as

$$E\left(\ln y_i^H | S_i^H = 1\right) = \rho_0 + \rho_1 + E\left(\epsilon_{Hi} | S_i^H = 1\right), \tag{9}$$
$$E\left(\ln y_i^H | S_i^H = 2\right) = \rho_0 + 2\rho_1 + E\left(\epsilon_{Hi} | S_i^H = 2\right), \tag{10}$$
$$Var\left(\ln y_i^H\right) = \rho_1^2 Var\left(S_i^H\right) + Var\left(\epsilon_{Hi} | m = 1\right). \tag{11}$$

Note that $E\left(\epsilon_{Hi}|S_i^H = 1, 2\right)$ and $Var\left(\epsilon_{Hi}|m = 1\right)$ depend on the marriage decision rule, equation (1). Therefore, they are functions of $\{P^0, P^1, a_0, a_1, \psi, \sigma_H\}$. In this model, the husband's earnings parameters can only be identified together with those parameters that determine marriage outcomes.[5] From equations (5) to (11), $\left\{P^0, P^1, \frac{a_0}{\psi}, \frac{a_1}{\psi}, \rho_0, \rho_1, \sigma_H\right\}$ can be identified. a_0, a_1, and ψ are not separately identified because the husband's earnings and marriage utility enter the individual utility function linearly. Finally, cs is identified from the attendance rate, $\Pi_S = F_w\left[\epsilon_w^*\left(cs, \beta_0, \beta_1, \sigma_w, P^0, P^1, \frac{a_0}{\psi}, \frac{a_1}{\psi}, \rho_0, \rho_1, \sigma_H\right)\right]$, where cs is the only unknown variable.

Next let us consider a heterogeneous sample with two types. Using a similar argument, $\left\{P^0, P^1, \frac{a_0}{\psi}, \frac{a_1}{\psi}, \rho_0, \rho_1, \sigma_H\right\}$ are identified from the marriage distribution and the husband's earnings. Let $\beta_0(\delta_k) = \beta_{0k}$, $k = h, l$, and $\epsilon_{wk}^* = \epsilon_w^*\left(\theta^-, \beta_{0k}\right)$, where θ^- includes all the parameters except for β_0. Compared with the homogeneous case, additional moments are used to identify type-specific β_0 and type proportion π. The college attendance rate is now the weighted average of the attendance rates of both high and low skill types:

$$\Pi_S = \pi F_w\left(\epsilon_{wh}^*\right) + (1 - \pi) F_w\left(\epsilon_{wl}^*\right). \tag{12}$$

At $t = 1$, earnings are observed only for those who choose not to attend college. Therefore,

$$E(\ln y_1) = \pi \int_{\epsilon_{wh}^*}^{\infty} (\beta_{0h} + \beta_1 + \epsilon_w) f(\epsilon_w) d\epsilon_w + (1 - \pi) \int_{\epsilon_{wl}^*}^{\infty} (\beta_{0l} + \beta_1 + \epsilon_w) f(\epsilon_w) d\epsilon_w. \tag{13}$$

At $t = 2$, the mean of high school graduates' earnings is the weighted average of mean earnings of both types.

$$\begin{aligned} E(\ln y_2 | S = 1) &= \pi_1 E(\beta_{0h} + \beta_1 + \epsilon_w) + (1 - \pi_1) E(\beta_{0l} + \beta_1 + \epsilon_w) \\ &= \pi_1(\beta_{0h} + \beta_1) + (1 - \pi_1)(\beta_{0l} + \beta_1). \end{aligned} \tag{14}$$

[5] This is the standard argument for selection models for the identification of the wage offer parameters. If all potential husbands' earnings are observed, $E\left(\epsilon_{Hi}|S_i^H = 1\right) = E\left(\epsilon_{Hi}|S_i^H = 2\right) = 0$ and $Var\left(\epsilon_{Hi}|m = 1\right) = \sigma_H^2$, so ρ_0, ρ_1, and σ_H are identified from an OLS regression.

The earnings variance of high school graduates is

$$Var\left(\ln y_2|S=1\right)=\pi_1\left(1-\pi_1\right)\left(\beta_{0h}-\beta_{0l}\right)^2+\sigma_w^2, \tag{15}$$

where $\pi_1=\frac{\pi\left(1-F_w\left(\epsilon_{wh}^*\right)\right)}{1-\Pi_S}$ is the proportion of high skilled women among high school graduates. The mean and variance of earnings of college graduates are similarly determined.

$$E\left(\ln y_2|S=2\right)=\pi_2\left(\beta_{0h}+2\beta_1\right)+\left(1-\pi_2\right)\left(\beta_{0l}+2\beta_1\right), \tag{16}$$

$$Var\left(\ln y_2|S=2\right)=\pi_2\left(1-\pi_2\right)\left(\beta_{0h}-\beta_{0l}\right)^2+\sigma_w^2, \tag{17}$$

where $\pi_2=\frac{\pi F_w\left(\epsilon_{wh}^*\right)}{\Pi_S}$ is the proportion of high skilled women among college graduates. Equations (12) to (17) identify the parameters $\{\pi, cs, \beta_{0h}, \beta_{0l}, \beta_1, \sigma_w\}$.[6] The moment conditions incorporate the selection rules predicted by the model and the functional form assumptions on the distributions of earnings and unobserved heterogeneity. This is the standard argument for the identification of selection models (Heckman 1979).

[6] For a homogenous sample, equations (16) to (17) become

$$E\left(\ln y_2|S=1\right)=\beta_0+\beta_1,$$
$$E\left(\ln y_2|S=2\right)=\beta_0+2\beta_1,$$
$$Var\left(\ln y_2\right)=\sigma_w^2.$$

Therefore, a simple *OLS* regression on second-year wages will identify β_0, β_1, and σ_w.